Reforming Estate Taxation by Reversing the Generation-Skipping Transfer Tax*

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Abstract

Although the existing U.S. code, through the Generation-Skipping Transfer Tax (GSTT), levies a higher tax on estates passed directly to grandchildren compared to estates passed to adult children when adult children are still alive, previous work on the design of estate taxation has ignored the welfare effects of estate-tax rates that depend on the age of heirs. To provide a proof of concept, we construct a two-period overlapping-generations model with differential taxation of estates based on the age (birth generation) of heirs, and we examine how equilibrium welfare is affected as the estate-tax rate is levied differentially on bequests passed to grandchildren versus bequests passed to adult children. In numerical simulations of the model, as in reality, the estate tax does not generate a large share of total tax revenue, relative to the labor-income tax. Nevertheless, while preserving revenue neutrality, we find that welfare can be improved if the estate-tax rate paid on estates passed to grandchildren is lower than the rate paid on estates passed to adult children, in contrast to the existing tax code. The improvement of welfare via a reversal of the GSTT in the model results from the fact that lifetime resources have a higher present value in equilibrium. The welfare improvement is magnified as the tax on estates passed to adult children is monotone increasing, while adjusting the labor-income tax accordingly to preserve revenue neutrality. We highlight that removing (or at least reducing) the tax rate on estates passed to grandchildren compared to estates passed to adult children is a proxy example of a reverse social security program (i.e., the transfer of resources from older generations to younger generations) that improves equilibrium welfare in an economy that is dynamically efficient, a concept noted by Samuelson (1975) without example.

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1 Introduction

A tax on estates, sometimes referred to by its detractors as "the death tax," is often viewed as a controversial tax instrument. Researchers have given some attention to the optimal structure of the estate tax, which might be to have no estate tax at all. On the one hand, an estate tax can be viewed as a tax on capital, which many researchers argue should be eliminated since it reduces the return on capital ((Foster and Fleenor, 1996; Johnson and Eller, 1998)). But on the other hand, an estate tax can be implemented to reduce inequality and thereby increase ex ante expected lifetime utility ((Piketty and Saez, 2013; Saez and Zucman, 2019)). In support of this idea, Piketty et al. (2011) argue for an estate tax rate of 50% or more.[1]

Two questions that researchers have largely ignored are: 1.) What is optimal bequest policy? and, 2.) How might the estate tax be employed to achieve such a normative objective? To our knowledge, the only study that attempts to address the first issue is Feigenbaum et al. (2013). In a model with no exogenous bequest motive, they document that if bequests are given exclusively to the very young, then a generalized market equilibrium exists in which lifetime utility approaches the maximal feasible utility of the golden-rule allocation, building on the primary concept explored in Feigenbaum et al. (2011). This far exceeds the lifetime utility of the rational competitive equilibrium under the standard assumption that bequests are distributed uniformly across the surviving population.

We focus on the second question in this study. Namely, what happens to lifetime utility if the bulk of a decedent's estate is transferred to grandchildren (younger heirs), as opposed to adult children (older heirs)? We find that lifetime welfare can be increased! Suppose that the estate tax rate is designed to be a function of the age of the heir. If the tax rate on an estate passed to younger heirs is lower than the tax rate on an estate passed to older heirs, then wills ought to be revised in order to shift gifts from older friends and relatives to younger friends and relatives.[2] In fact, the existing tax code in the United States already taxes an estate that is passed directly to grandchildren differentially compared to an estate that is passed to adult children. Specifically, the Generation-Skipping Transfer Tax (GSTT) was designed

[1] Kopczuk (2013) gives an overview of the theoretical models and the empirical evidence of the redistributive role of the taxation of wealth in the form of intergenerational transfers, in particular, estate taxation. Among the most recent literature, Gravelle (1997), Gale and Perozek (2001), and Gale and Slemrod (2001a) provide overviews and discussions on estate and gift taxation.

[2] Yoo (2012) similarly argues that the gift tax should be eliminated to encourage more inter vivos transfers from the old to the young.

and implemented to discourage the passing of an estate directly to grandchildren, instead of first passing it to the surviving adult children of the decedent. The underlying rationale for the creation of the GSTT is to discourage the bequeathing of an estate to grandchildren as a tax avoidance strategy, bypassing an additional incidence of estate taxation in which the surviving estate would be taxed when it is passed from the adult children of the original decedent to their children (who are the grandchildren of the original decedent). The design and implementation of the GSTT implies a normative objective imposed by policymakers: an estate ought to be taxed twice on its way to grandchildren; once when it is passed from a decedent to their surviving adult children, and then once again when it is passed from adult children to their own children, who are the grandchildren of the original decedent (see Abrams (1987), Plant and Wintriss (1988), and Brunson (2019) for an overview of this normative objective that underlies the creation of the GSTT).

In this paper we provide a proof of concept that this normative objective of generational double taxation, embodied as the Generation-Skipping Transfer Tax, might undermine a competing normative objective, namely the use of tax policy as an instrument to improve the lifetime welfare of individuals.[3] In our modeling framework, the existence of the GSTT leads to lower lifetime welfare, and lifetime welfare can be improved by reversing the GSTT. If the estate of a decedent passes directly to grandchildren, bypassing surviving adult children, then lifetime resources have a higher present value in equilibrium (which improves possibilities and welfare). Moreover, the inheritance of an estate by grandchildren allows them to save their inherited wealth over a much longer time span, which can have favorable attributes in equilibrium, like a larger capital stock which improves both wages and welfare.

At least since Samuelson (1975), it has been known that implementing an intergenerational transfer program like social security improves steady-state welfare in a dynamically efficient competitive equilibrium *only if* the program works in reverse: transferring wealth from older generations to younger generations, instead of what is done in real-world social security programs... transferring wealth from younger generations to older generations. Economists typically find the implementation of a reverse social security program to be problematic when evaluated through the lens of the Pareto Criterion. Namely, such a program

[3]The large body of existing research on the Generation-Skipping Transfer Tax predominantly discusses how to best conduct financial planning and estate planning for tax avoidance purposes given the existing structure of the GSTT (e.g., see Fellows (2006); Gallo (1998); Haneman (2022); Jackson (1977); McCaffery (2023)), as opposed to studying the higher-order question of how the design of the GSTT might achieve or undermine normative objectives. We focus on a higher-order question how welfare is affected by the design of the tax instrument.

cannot be implemented in a Pareto-improving way given that the initial older generation would incur a negative transfer and reduction in lifetime resources without ever having received any transfer benefit when they were younger.[4] An age-dependent estate tax structure that encourages the passing of some of an estate from a decedent directly to grandchildren (reversing the GSTT) acts very much like the principle of a reverse social security program, except that its approval and implementation ought to be more palatable from a political perspective, given that it is a windfall gain to grandchildren (surviving adult children could also receive some of the inheritance) *and* the decedent, by definition, is deceased!

As our proof of concept to document the potential welfare gains from the existence of an age-dependent estate tax, we construct a two-period overlapping generations model in which an household chooses to pass their estate to their adult children (the generation behind them) and/or to their grandchildren (the next subsequent generation). We assume that households in the model possess a "warm glow" bequest motive following De Nardi (2004), in which a household's utility depends directly on the amount bequeathed to heirs, as opposed to household utility depending altruistically on the utility of heirs that is embedded in a progenitor's utility function (Barro, 1974). In numerical simulations and exercises, we document that equilibrium lifetime welfare is monotone decreasing as the ratio of estate tax rates on grandchildren to adult children increases.[5] And even though the current amount of collected estate taxes constitute just a small fraction of total tax revenues collected by the U.S. federal government in reality, the point of our proof of concept is that a reversal of the GSTT might proxy a reverse social security program with the possibility of associated welfare gains, especially if exemptions on what constitutes a taxable estate are also revisited and potentially revised.

[4]Moreover, acquiring political approval to create and implement a reverse social security program would face non-trivial difficulty, given that older generations vote in much higher proportions during elections or referenda compared to younger generations.

[5]One issue that would need to be addressed in any type of real-world implementation is how to ensure that an estate bequeathed to grandchildren actually ends up with them and not with their parents. Presumably, this problem could be solved by requiring that inheritances be placed into suitably administered trusts that grandchildren would not be able to access until adulthood. We abstract from this issue in the context of our two-period overlapping generations model given that grandchildren do not receive an inheritance until they become adults in the model.

2 A Few Facts about Estate Taxation in the U.S.

The estate tax in the United States is a federal tax on the transfer of an estate from a person who dies (a decedent) to a survivor (an heir). The tax applies to property that is transferred by will, or if the person has no will, is transferred according to state laws of intestacy. Other transfers that are subject to the tax can include those made through a trust and the payment of certain life insurance benefits or financial accounts. The estate tax is part of the federal unified gift and estate tax code in the United States. The estate and gift tax, enacted in 1916, is the only wealth tax levied by the U.S. federal government. Over the years, the federal estate tax has undergone a number of changes, including changes to the exclusion amount, the tax rate structure, and the definition of what constitutes a taxable estate.[6] Because of exemptions in the tax code, it is estimated that only the largest 0.2% of estates in the U.S. pay the estate tax.[7] In 2017, the estate-tax exemption was $5.49 million, and the exemption doubled in 2018 to $11.18 million as a result of passage of the Tax Cuts and Jobs Act of 2017. Tax revenues collected from the estate and gift tax are relatively small, as is the fraction of estates that pay estate taxes: only 2 percent of the estates of adult decedents pay any estate taxes and tax revenue collected is about 0.3 percent of U.S. output (see, for example, Gale and Slemrod (2001b)).

One feature of the estate-tax code in the United States is a tax called the "Generation-Skipping Transfer Tax" (GSTT), which is a tax on gifts or bequests to a beneficiary or heir who is two or more generations younger than the transferor or decedent.[8] For example, a gift or bequest is considered to be "generation-skipping" if it is transferred from a grandparent to a grandchild, or from a great-aunt to a great-nephew. Younger beneficiaries or heirs are known in tax parlance as "skip persons", meaning that a beneficiary or heir is considered to be a "skip person" if they are related by blood, by marriage, or by adoption, *and* is the transferor's or decedent's grandchild, grandniece/grandnephew, or first cousin twice removed (or great-grandchild, great-grandniece/grandnephew, first cousin thrice removed, etc.). The GSTT tax rate is the same as the maximum estate-tax rate, which is 40 percent currently.[9]

[6]For example, an estate in considered taxable at $5.34 million for decedents in 2014 and $5.45 million in 2015 (effectively $10.90 million per married couple, assuming the deceased spouse did not leave assets to the surviving spouse) for estates of persons dying in 2016. "What's New – Estate and Gift Tax". www.irs.gov

[7]Huang, Chye-Ching; DeBot, Brandon. "Ten Facts You Should Know About the Federal Estate Tax". Center on Budget and Policy Priorities.

[8]The Generation-Skipping Transfer Tax was enacted by section 2006 of the Tax Refom Act of 1976, Pub. L. No. 94-455, 90 Stat. 1879-90 (1976).

[9]I.R.S. section 2641(a)(1).

3 The Model

We consider an overlapping-generations (OLG) model in which agents live for at most two periods, which we refer to as the young and the old. When an old agent dies, he can leave his wealth to members of the ensuing old cohort, who we interpret as the agent's children, or to members of the ensuing young cohort, who we interpret as the agent's grandchildren.

To be precise, in each period a new cohort of population 1 is born. Let Q_s be the probability of surviving till age s, where $1 = Q_0 \geq Q_1 > Q_2 = 0$. We consider only steady-state equilibria, so macroeconomic variables such as the wage w are time-independent. Every young agent supplies n in the labor market for w. This labor income is subject to the tax θ^l.

Consumption at age s is c_s. Let a_{s+1} be the saving of a household at age s, which will earn the gross rate of return R. The household writes a will that apportions how much of any assets will be given to each surviving cohort in the event of the household dying as planned at age 2. Let b_s be the assets distributed to a household of age s if the deceased household lives a full life.[10] A bequest by a household of age s is subject to the estate tax θ^e_s.

We assume the utility that households gain from leaving bequest (b_0, b_1) has this form

$$H(b_0, b_1) = \begin{cases} R[(1-\theta^e_0)b_0]^{\mu_0}[(1-\theta^e_1)b_1]^{\mu_1} & \zeta = 1 \\ R\left[\mu_0((1-\theta^e_0)b_0)^{1-\zeta} + \mu_1((1-\theta^e_1)b_1)^{1-\zeta}\right]^{\frac{1}{1-\zeta}} & \zeta \neq 1 \end{cases} \tag{1}$$

be a CES aggregator with elasticity of substitution ζ^{-1}, where $\zeta > 0$, and $\mu_s \geq 0$ is the relative strength of the bequest motive for a household of age s. We also normalize $\mu_0 + \mu_1 = 1$. Note that (1) embeds the effects of taxes and the rate of return in the aggregator H, so the household's preferences depend on the after-tax bequest that its heirs will actually receive rather than just the funds it sets aside for them.

Finally, we define B_s to be the bequest received by a household at age s. and the period utility function, $u(x)$ is a CRRA utility function with risk aversion $\gamma > 0$

[10]The will does not apply to households that die prematurely because the bequest motive would impose a borrowing constraint that complicates the solution of the model.

Household problem, Let $\beta > 0$ be the internal discount factor and $\rho > 0$ be the discount factor associated with bequests. The household maximizes

$$U = u(c_0^{\eta} l^{1-\eta}) + \beta\eta Q_1 \left[u(c_1) + \rho u(H(b_0, b_1))\right] \tag{2}$$

subject to

$$c_0 + a_1 = (1 - \theta^l)w(1 - l_0) + Ra_0 + B_0 \tag{3}$$

$$c_1 + a_2 = Ra_1 + B_1 \tag{4}$$

$$b_0 + b_1 = a_2 \tag{5}$$

$$b_s \geq 0 \quad s = 0, 1 \tag{6}$$

$$a_0 = 0. \tag{7}$$

where c_s is the consumption at age s, l_0 is the leisure for young agents. We assume each agent has 1 unit of time at age 0 and therefore the labor supply will be equal to $1 - l$.

Here, we use a "warm glow" bequest motive as in De Nardi (2004) in which utility is derived from the size of the bequest (after taxes) as opposed to the utility of the heir, though we use a different specification of the bequest utility function than De Nardi so we can obtain analytic results. Note that we assume there is only a bequest motive in the event that the household lives a full life.[11]

Production side, on the production side of the economy, there is a firm with a Cobb-Douglas production function that produces output

$$Y = K^{\alpha} N^{1-\alpha}, \tag{8}$$

where N is the labor supply

$$N = 1 - l, \tag{9}$$

and K is the capital stock

$$K_t = a_1 + Q_1 a_2. \tag{10}$$

[11] This simplification allows us to avoid imposing borrowing constraints. As in Feigenbaum and Gahramanov (2012), we ignore the fact that debts will not be passed on to heirs in reality.

Factor prices are then

$$w = w(K) \equiv (1-\alpha)\left(\frac{K}{N}\right)^{\alpha} \tag{11}$$

and

$$R = R(K) \equiv \alpha\left(\frac{K}{N}\right)^{\alpha-1} + 1 - \delta \tag{12}$$

Since there is no bequest motive for agents who die prematurely, we assume their assets are distributed uniformly.

Considering the survival probability structure, the total population is

$$P = 1 + Q_1$$

Using that we can form the bequest balance equations as in the following

$$Q_s B_s = (1-\theta_s^e) R \left[\frac{Q_s}{P}(1-Q_1)a_1 + Q_1 b_s\right] \tag{13}$$

for $s = 0, ..., T$.

Government, government's total tax revenue in this economy will be the sum of estate tax revenue and labor tax revenue. Estate tax revenue can be calculated as

$$\Theta^e = R\sum_{s=0}^{1}\theta_s^e\left[\frac{Q_s}{P}(1-Q_1)a_1 + Q_1 b_s\right] = \sum_{s=0}^{1}\frac{\theta_s^e}{1-\theta_s^e}Q_s B_s, \tag{14}$$

in which the second part follows from the bequest balance:

$$B_s = (1-\theta_0^e) R b_s \tag{15}$$

Labor tax revenue is

$$\Theta^l = \theta^l w N. \tag{16}$$

Finally, the government's budget constraint is

$$G = \Theta_t^e + \Theta_t^l, \tag{17}$$

where G is exogenous government spending.

Equilibrium; in this environment, a steady-state competitive equilibrium consists of a consumption allocation (c_0, c_1), leisure level (l_0) asset holdings (a_1, a_2), planned bequests (b_0, b_1), a capital stock K, actual bequests (B_0, B_1), factor prices R and w, estate tax rates (θ_0^e, θ_1^e), and tax revenues Θ^e and Θ^l such that

(i) the consumption allocation, asset holdings, and planned bequests maximize (2) subject to (3)-(7) given the factor prices, actual bequests, and tax rates;

(ii) the factor prices satisfy (11) and (12) given the capital stock;

(iii) the asset holdings are consistent with (10) given the capital stock;

(iv) the estate tax revenues satisfy (14) given the interest rate, estate tax rates, planned bequests, and asset holdings;

(v) the labor tax revenues satisfy (16) given the labor tax rate and wage;

(vi) the actual bequests satisfy (15) given the interest rate, estate tax rates, asset holdings, and planned bequests;

(vii) government spending satisfies (17) given the interest rate and tax revenues.

4 The Analytic Case

While we have kept the model relatively simple, including only what we need to separately consider taxes on estates earned by children versus grandchildren, it is still complicated enough to be fairly opaque about the cause of our main results. To further explicate our findings, in this section we will consider a special case that yields analytic results.

In the case where we assume $\gamma = \delta = \eta = \zeta = Q_1 = 1$, labor is supplied exogenously as $N = 1$. The other policy rules simplify to be proportional to the lifetime wealth

$$W = (1 - \theta_l)w + B_0 + \frac{B_1}{R}. \tag{18}$$

The marginal propensity to consume when young out of lifetime wealth is

$$m_0 = \frac{1}{1 + \beta(1 + \rho)} \in [0, 1]. \tag{19}$$

Then we have

$$c_0 = m_0 W, \tag{20}$$

$$c_1 = \beta R m_0 W, \tag{21}$$

$$b_0 = \mu_0 \beta \rho R m_0 W, \tag{22}$$

and

$$b_1 = \mu_1 \beta \rho R m_0 W. \tag{23}$$

Note that the main reason why the model is so simple in this special case is that the policy rules do not depend on the estate tax rates when $\eta = 1$. This eliminates the government's ability to encourage households to leave their wealth to their grandchildren rather than their children. However, it still maximizes social welfare for the government to extract revenue by taxing bequests and children and to eliminate taxes on bequests to grandchildren.

In this case, lifetime utility is

$$\begin{aligned} U &= \ln(c_0) + \beta\left[\ln(c_1) + \rho\left[\ln R + \mu_0 \ln\left((1-\theta_0^e)b_0\right) + \mu_1 \ln((1-\theta_1^e)b_1)\right]\right] \\ &= (1+\beta+\beta\rho)\ln W + \beta\rho\left[\ln R + \mu_0 \ln(1-\theta_0^e) + \mu_1 \ln(1-\theta_1^e)\right] + U_0, \end{aligned} \tag{24}$$

where U_0 is a function of exogenous variables.

Although we have three tax rates in this special case, if we fix G we can express the endogenous variables as functions of θ_1^e alone. That is to say, the labor tax and the tax on estates inherited by young households are perfect substitutes. Since the utility function (24) depends on θ_0^e but not θ^l, it is never optimal to tax estates inherited by young households unless $\theta_l = 1$. Of course, this is an artificial consequence of assuming labor is supplied inelastically, so in the following we will fix both G and θ_l, which is reasonable since the point of the current exercise is to focus on the tradeoff between taxing estates inherited by the young versus the old, not the tradeoff between labor taxes and estate taxes.

Proposition 1. *If $\gamma = \delta = \eta = \zeta = Q_1 = 1$, given tax rates θ_l, θ_0^e, and $\theta_1^e \in [0,1]$, there is a unique steady state equilibrium with gross interest rate*

$$R = \frac{-B_R - \sqrt{B_R^2 - 4A_R C_R}}{2A_R}, \tag{25}$$

that solves

$$A_R R^2 + B_R R + C_R = 0, \tag{26}$$

where

$$\begin{aligned} C_R = \sigma_l &= \frac{1}{1-\theta^l}\frac{\alpha}{1-\alpha} > 0 \\ B_R &= -[1 - m_0 + \beta\rho m_0 \sigma_l \nu_1] < 0 \\ A_R &= -m_0\beta\rho[1 - \nu_1 + \sigma_l \nu_0] < 0. \end{aligned}$$

The following proposition establishes the condition under which increasing θ_1^e and reducing θ_0^e while holding G and θ^l constant in general equilibrium will result in a smaller interest rate and thus a larger capital stock, output, and before-tax wage.

Proposition 2. *If $\gamma = \delta = \eta = \zeta = Q_1 = 1$, given tax rates θ_l, θ_0^e, and $\theta_1^e \in [0, 1]$, we have the comparative statics results*

$$\frac{\partial R}{\partial \theta_0^e} = -\frac{m_0\beta\rho\sigma_l\mu_0 R^2}{2RA_R + B_R} > 0 \tag{27}$$

and

$$\frac{\partial R}{\partial \theta_1^e} = \frac{m_0\beta\rho\mu_1 R(R - \sigma_l)}{2RA_R + B_R}. \tag{28}$$

The latter derivative is of ambiguous sign but will be negative if $R > \sigma_l$.

Thus if the tax on estates inherited by old households is increased and the tax on estates inherited by young households is decreased so as to keep tax revenue constant, the interest rate will decrease and the capital stock will increase if the interest rate is sufficiently large.

Note that for a typical calibration of the production function with a share of capital on the order of 1/3, $\frac{\alpha}{1-\alpha}$ will be on the order of 1/2. Thus the labor-tax rate would need to be more than about 50% in order for σ_l to be greater than 1. Assuming we do not have that, the condition that $R > \sigma_l$ will be less stringent than the condition $R > 1$ for dynamic efficiency that commonly appears in propositions of this sort. If this usually looser condition is satisfied, we will have that R increases with the young inheritance tax rate and decreases with the old inheritance tax rate.

The effect on welfare will be more complicated since the bequest utility depends on the after-tax bequest rather than the before-tax bequest, so the effect of a higher θ_1^e on capital may be offset by the loss of utility via the bequest motive. In Section 6, we show for a typical calibration that shifting the inheritance tax from young heirs to old heirs does increase welfare.

5 Calibration

We can view the mortality profile $\{Q_s\}_{s=0}^T$, and the share of capital as observable parameters. As unobservable parameters, we have β, γ, η, ρ, and $\{\mu_1\}_{s=0}^T$. We also have tax rates $\theta_0^e, ..., \theta_T^e$ and θ_l. Then we need to calibrate the model to match K/Y, Θ^e/Y, G/Y, D/Y, the ratio of consumption to bequests for the elderly c_1/a_2, and the relative size of bequests

$$\frac{B_s}{\sum_{i=0}^T B_i}$$

to each age group for $s = 0, ..., T$. As is well known, we cannot separately identify β and γ from steady-state variables. We also will not be able to separately identify the ρ and ζ from steady-state variables. We will follow the strategy of choosing γ and ζ and choosing β and ρ to fit our calibration targets. Since we are setting $T = 1$, δ as measured for a period will be close to 1 regardless of what it is at the annual rate, so we set $\delta = 1$ rather than try to match C/Y.

Recall that ζ^{-1} is the elasticity of substitution between bequests at different ages. It seems reasonable that this elasticity ought to be very high since most bequests are given to children and not to grandchildren, though it is doubtful that children would be given significantly more weight than grandchildren in the utility function.[12] However, if ζ is close to zero, then a slight increase in the weight of children over the weight of grandchildren will lead to most bequests going to children.

We calibrate the model with $T = 1$ so when the elderly population dies they leave their wealth either to the subsequent age-1 generation, i.e. their children, or to the subsequent age-0 generation, i.e. their grandchildren. We view a period as being 30 years long. If we set $K/Y = 3$ in annual terms, we then have $K/Y = 0.1$ in period terms. To calibrate Θ^e/Y we use data from historical table at the white house[13] we calculated $\Theta^e/Y = 0.0025$. With a typical depreciation rate of 8-10% nearly all capital will have depreciated after thirty years, so let us set $\delta = 1$ on a per period basis. We also set the share of capital $\alpha = 1/3$.

[12]Indeed, evolutionary arguments suggest that, if anything, the weight on grandchildren ought to be higher than the weight on children.

[13]https://www.whitehouse.gov/omb/budget/historical-tables/

Since we are imagining the two periods as going from ages 25 to 55 and then 55 to 85, let us set $e_0 = 1$ and $e_1 = 1/3$. From mortality tables, we set $Q_0 = 1$ and $Q_1 = 0.92$. Calibrating the model to current data, we set $\theta_0^e = \theta_1^e = \theta^e$. We choose θ^e to match Θ^e/Y. For simplicity, we also assume a balanced budget and set $D = 0$ for now. Then $G = \Theta^e + \Theta^l$. If we calibrate $G/Y = 0.15$, we need

$$\frac{\Theta^l}{Y} = \frac{G}{Y} - \frac{\Theta^e}{Y} = 0.15 - 0.0025 = 0.1475.$$

Since

$$\frac{\Theta^l}{Y} = \frac{\theta^l wN}{Y} = (1-\alpha)\theta^l,$$

we need

$$\theta^l = \frac{1}{1-\alpha}\frac{\Theta^l}{Y} = \frac{3}{2}0.1475 = 0.22125.$$

For the baseline model, let us suppose $\gamma = 1$ and $\eta = 1/3$. Hurd and Smith (2002) report that $\frac{c_1}{y_1+Ra_1} = 0.6$ for households between 70 and 75. considering the total wealth (18) and our choice for consumption function, we have

$$\frac{c_1}{y_1 + Ra_1} = \frac{1}{1+\rho}.$$

Thus we should have

$$\rho = \frac{1}{\frac{c_1}{y_1+Ra_1}} - 1 = \frac{5}{3} - 1 = \frac{2}{3}.$$

Since there is very little early mortality, planned bequests ought to dominate bequests.

$$\frac{b_0}{b_0 + b_1} = \frac{1}{1+\mu_1^{1/\eta}}.$$

For our numerical exercises here, let us assume grandchildren receive a sixth of bequests, so $b_0/b_1 = 0.2$. Then

$$\mu_1 = \left(\frac{b_1}{b_0}\right)^{\eta},$$

so we set $\mu_1 = 1.72$. Finally, we set β to match K/Y.

6 Discussion: Welfare Impact of Estate Tax - Numerical Insights

After exploring the analytical framework of our model, where we delineated specific propositions under certain functional forms, we now turn our attention to numerical experiments. These are designed to provide deeper insights and complement the intuitions derived from the analytical case. Importantly, these experiments incorporate an elastic labor supply to offer a more nuanced understanding of how tax structure changes influence labor behavior.

In our first experiment, we aim to investigate the effects of removing the generation-skipping clause from the estate tax structure. Specifically, we equalize the estate tax rate for both the young and old generations in the model, denoted as $\tilde{\theta}^e$. We keep the labor income tax unchanged in this exercise. To ensure comparability, we calibrate the uniform estate tax rate such that the government's tax revenue remains at the level observed in the benchmark economy. Building on the initial experiment, we then pivot to explore the implications of redistributing the estate tax burden exclusively to one generation at a time. In one scenario, the estate tax is entirely shouldered by the old generation, i.e. the children, while it is entirely removed for the young generation, or grandchildren. In an alternate scenario, we flip this arrangement, removing the estate tax for the old generation and applying it solely to the young generation. In both of these redistributive exercises, the estate tax rates are calibrated to ensure that the government's tax revenue remains at the level observed in the benchmark economy. Finally, in a last experiment, we remove the estate tax for both generations and find the labor income tax to keep the government's tax revenue constant. This exercise aims to gauge the extent to which estate taxation is distortionary when compared to other forms of income taxation. The primary objective of these numerical experiments is to assess the welfare impact of different estate tax structures, specifically focusing on the effects of generation-specific tax burdens.

6.1 Uniform estate tax across generations

In the first numerical experiment, we undertake an analysis of an alternative steady-state economy where the estate tax is stripped of its generation-skipping feature. In this simulated setting, a uniform estate tax rate is imposed on both the young and old generations of heirs, i.e. children and grand children, denoted as $\theta_0^e = \theta_1^e = \tilde{\theta}^e$. Through an iterative process, the tax rate $\tilde{\theta}^e$ was calculated to ensure that the government's revenue remains consistent with its level at the benchmark economy.

The findings from this experiment are laid out in Table 1 and compared with the benchmark economy for a more nuanced understanding. As seen in the data presented in the "untifor estate tax" column, the act of equalizing the estate tax rate across generations leads to notable welfare gains for the economy's agents. Specifically, while the old generation experiences a marginal uptick in their estate tax obligation, the young generation enjoys a substantial benefit from a reduced tax rate, established at $\tilde{\theta}^e = 1.17\%$.

This decrease in the estate tax for the young generation effectively increases the capital they inherit at the start of their lives, thereby enhancing its lifetime wealth. This increase in initial capital is reflected in the elevated investment-to-output ratio, suggesting a heightened propensity for the young generation to save and invest. We do have a marginal decline in labor supply, which can be attributed to the income effect induced by the increased initial capital. However, the overall positive welfare gains indicate that the advantages of receiving higher initial capital more than compensate for this reduction in labor supply. Given that the government's tax revenue is not adversely affected by this tax structure change, the experiment provides compelling evidence that the generation-skipping feature does not offer any inherent advantages in the context of the estate tax structure.

6.2 Tax-free estate for young generation

Following our initial examination of a uniform estate tax, the second experiment takes a different approach by exempting the young generation from the estate tax entirely while imposing it solely on the old generation. This is, in essence, a reversal of the traditional generation-skipping tax structure, where the burden usually falls on the younger heirs. Specifically, we set the estate tax rate for the young generation, θ_0^e, to zero and iteratively determine the old generation's estate tax rate θ_1^e that yields the same level of government tax revenue as in the baseline scenario.

The outcomes of this experiment are detailed in the forth column of Table 1. When evaluating the steady state of this modified tax structure against the benchmark economy, a nuanced picture emerges. We note a decrease in labor supply by precisely 0.02 percentage points, a trend that mirrors what was observed in the first experiment. This decline can largely be ascribed to the income effect instigated by the tax alterations. With the young generation no longer burdened by an estate tax, they start their adult lives with an increased amount of capital relative to the benchmark economy. This spike in initial capital endows them with higher lifetime wealth, a change that manifests itself in altered labor supply behaviors. This increased initial wealth is also evident in the increase in investment-to-output ratio, indicating a shift in savings and investment behavior among the young generation.

Table 1: Welfare Implications of Alternative Estate Tax Structures: Numerical Experiment Results

Panel A: Imposed tax rates

	Benchmark	Uniform Estate tax	Tax-free Estate for Young	Tax-free Estate for Old	Tax-free Estate for All
θ_0^e	2%	$\tilde{\theta}^e = 1.17\%$	0	$\tilde{\theta}_0^e = 6.9\%$	0
θ_1^e	1%	$\tilde{\theta}^e = 1.17\%$	$\tilde{\theta}_1^e = 1.4\%$	0	0
θ^l	30%	30%	30%	30%	$\tilde{\theta}^l = 30.4\%$

Panel B: Steady states comparison

	Benchmark	Uniform Estate tax	Tax-free Estate for Young	Tax-free Estate for Old	Tax-free Estate for All
Capital	100	100.09	100.23	99.41	99.22
Output	100	100.02	100.05	99.87	99.45
Labor	100	99.98	99.97	100.06	99.71
$\frac{C}{Y}$	100	99.98	99.97	100.05	99.96
$\frac{I}{Y}$	100	100.06	100.15	99.61	99.47
EV	.	+0.026%	+0.065%	-0.16%	-0.30%

Note: This tables summarizes the results of the experiments that are explained in the discussion section of the text. $\tilde{\theta}$ shows the tax rate calculated to keep the government revenue constant at its level in the benchmark economy.

Moreover, the 0.09 percentage points increase in capital leads to a 0.02 percentage point increase in output. When compared to the first experiment, where estate taxes were equalized but not eliminated for the young, the impact on capital and investment is more pronounced in this scenario. This aligns with the idea of reversing the social security system by facilitating a transfer from the old to the young, a move identified as welfare-improving in existing literature.

6.3 Tax-free estate for old generation

In this third experiment, we explore an alternative tax structure that amplifies the traditional generation-skipping transfer tax to its most extreme form. In this configuration, the old generation is completely exempt from the estate tax, and the entire tax burden is levied on the young generation. This setup serves as a drastic extension of typical estate tax arrangements.

To preserve a constant level of tax revenue, we calculate the estate tax rate for the young generation that would yield the same revenue as the benchmark scenario. Specifically, we set $\theta_1^e = 0$ for the old generation and determined $\theta_0^e = \tilde{\theta}^e$ for the young generation.

The findings of this third experiment are specifically detailed in the fifth column of Table 1. Contrary to our expectations, agents born into the steady state of this economy actually experience a welfare loss when compared to those in the baseline scenario. Interestingly, the estate tax rate for the young generation required to maintain consistent government revenue was calculated to be $\tilde{\theta}^e = 6.9\%$, a figure that represents a more than threefold increase over the baseline estate tax rate for this group.

This drastic increase in tax rate for the young generation manifests in various economic indicators. We observe an increase in labor supply, likely as a response to the heavier tax burden. Alongside this, there is a noticeable decline in both capital and the investment-to-output ratio. Given the increased level of the consumption-to-output ratio, it can be inferred that resources have been reallocated from saving to consumption, thereby explaining the reduced levels of aggregate output. Note that when we compare these results against the benchmark economy and the first two experiments, the significance of the old generation's contributions to estate tax revenue becomes increasingly evident

6.4 Eliminating estate tax

In our final experiment, we take the bold step of setting the estate tax rates for both young and old generations to zero. To compensate for this loss in tax revenue, we adjust the labor income tax rate, θ^l, to maintain a constant level of government revenue. The outcomes of this intriguing exercise are presented in the last column of Table 1. Unsurprisingly, the elevated level of labor income tax leads to a welfare loss, a finding that aligns with existing literature on the subject (seeDe Nardi (2004) as an example).

Upon examining various economic indicators, we find that the increase in labor income tax has a dampening effect on both labor supply and capital formation. This contraction in labor and capital subsequently results in a decline in the aggregate level of output, which in turn negatively impacts both consumption and investment levels. These dynamics help explain the welfare loss reported in the table for this experiment.

Notably, when we consider these results in the context of our previous experiments and the benchmark economy, an important inference emerges: estate taxes in our model appear to be relatively less distortionary compared to labor income taxes. This observation suggests a nuanced role for estate taxes in the overall tax policy landscape, potentially serving as a less disruptive alternative to labor income taxes for achieving revenue goals.

7 Avenues for Further Research

The present model is highly stylized by design in order to serve as a proof of concept as to how the Generation-Skipping Transfer Tax could actually reduce equilibrium welfare, in addition to reducing collected estate-tax revenue rather than increase it as intended by policymakers. We have parameterized the model to simplify the presentation, but there are two preference parameters that are likely to be particularly important in assessing the welfare effects of reversing the GSTT in numerical exercises: 1.) the elasticity of substitution between bequests to grandchildren (younger heirs) versus bequests to children (older heirs); and, 2.) the degree to which bequests exist as a luxury good in the model.

In our stylized model, we assume that bequests to children and bequests to grandchildren are neither complements nor substitutes, meaning that the elasticity of substitution is equal to unity. We make this assumption because of a lack of existing data that would, in principle, provide a vehicle with which to elicit and to identify an empirical value for this elasticity. Until an experiment comes along or is conducted in which the relative tax rates on bequests to children versus grandchildren is systematically adjusted, we can only venture a guess for the value of this elasticity.

Regarding whether bequests are a luxury good or not, we have employed the straightforward assumption of unit elasticity of substitution between consumption, leisure, and bequests. This assumption means that bequests do not serve as a luxury good in the model. We employ this assumption in order to acquire a fully analytic solution for the general equilibrium, by reducing the equilibrium condition into a single equation that can be solved numerically in a straightforward manner.[14] However, we point out that the possibility might exist to identify a sensible value for this elasticity parameter from data on the relationship between planned bequests and parental lifetime wealth, as outlined in De Nardi (2004).

As a segue from this last discussion point, the main shortcoming of the current framework is that we currently abstract from income and wealth heterogeneity across households of the same age. In reality, most households in the U.S. do not bequeath estates that are large enough to be subject to estate taxation. Because bequests can play a sizable role in shaping the wealth distribution (Kotlikoff and Summers (1981)), it would be an important feature in future work on this topic to include such types of heterogeneity. A richer modeling framework along these lines would enable a researcher to examine how estate tax reform, in general, might affect welfare for individuals at different points along the wealth distribution. Indeed, we speculate that it might be possible to adjust tax rates levied on estates passing to children versus grandchildren, while also reducing the labor tax in a revenue-neutral way, such that welfare can be improved *and* wealth inequality can be reduced.

8 Conclusion

The existing U.S. tax code levies a Generation-Skipping Transfer Tax (GSTT) on estates that are passed directly to grandchildren (younger heirs) as opposed to the surviving adult children (older heirs) of a decedent. The rationale for this tax is that the government loses out on estate-tax revenue when the estate of a decedent passes directly to grandchildren, instead of following the norm in which an estate passes to adult children, who will in their own time pass along the estate to their own children (i.e., to the grandchildren of the original decedent). In this study we construct a proof of concept to point out that this normative objective of policymakers, namely that an estate should be taxed twice on its way to grandchildren (once when it is passed from a decedent to surviving children, and then once again when the estate is passed from adult children to their own children) stands in opposition to another important normative objective: using tax policy as an instrument to improve the welfare of individuals (or, at a minimum, to avoid damaging the welfare of individuals). Indeed, we find that if the estate of a decedent passes directly to grandchildren, bypassing adult children, then lifetime resources have a higher present value in equilibrium, enhancing the

[14]This reduction of the equilibrium condition into a single equation for the capital stock occurs in general with the preferences of the household specified as (2).

choice set and improving the welfare of individuals. In addition, the inheritance of an estate by grandchildren allows them to save it over a much longer span of time, which can have favorable equilibrium outcomes like an increased capital stock, serving as an additional force to improve welfare.

To the best of our knowledge, the existing literature in economics and in public finance on estate taxation completely overlooks the idea of taxing the inheritances of children and grandchildren differentially. This is not surprising given that the GSTT is not that well known. In fact, almost all of the existing literature on the GSTT addresses the question of how to best structure and design an estate legally for tax avoidance purposes.

In providing our proof of concept as a first step in the literature on this topic, we construct a two-period overlapping-generations model to examine how revenue-neutral variation of the estate tax rate imposed on grandchildren (younger heirs) versus children (older heirs) can affect welfare in equilibrium. In numerical exercises of the model, we find that equilibrium lifetime welfare decreases as the ratio of the levied estate tax rates on grandchildren to children increases. And despite the fact that the current amount of collected estate taxes constitutes a small fraction of total tax revenues in the U.S., the take-home lesson of our proof of concept is that a reversal of the GSTT might proxy a reverse social security program (intergenerational transfers from the old to the young) with the possibility of associated welfare gains, an idea first pointed out by Samuelson (1975). The chances for this possibility would increases if exemptions on what constitutes a taxable estate are also revisited and revised.

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Appendices

A Income-Expenditure Identity

Define aggregate consumption as

$$C_t = \sum_{s=0}^{T} Q_s c_{s,t}.$$

Then

$$\begin{aligned}
C_t &= \sum_{s=0}^{T} Q_s[(1-\theta_t^l)w_t e_s + R_t a_{s,t} + B_{s,t} - a_{s+1,t+1}] \\
&= w_t(1-\theta_t^l)\sum_{s=0}^{T} Q_s e_s + R_t \sum_{s=0}^{T} Q_s a_{s,t} + \sum_{s=0}^{T} Q_s B_{s,t} - \sum_{s=0}^{T} Q_s a_{s+1,t+1} \\
&= w_t(1-\theta_t^l)N + R_t \sum_{s=0}^{T} Q_s a_{s,t} + R_t \sum_{s=0}^{T}(1-\theta_{s,t}^e)\left[\sum_{i=0}^{T-1} \frac{Q_s}{P}(Q_i - Q_{i+1})a_{i+1,t} + Q_T b_{s,t}\right] \\
&\quad -K_{t+1} - D_{t+1}
\end{aligned}$$

$$\begin{aligned}
C_t + G_t &= C_t + \Theta_t^e + \Theta_t^l + D_{t+1} - R_t D_t \\
&= (1-\theta_t^l)w_t N + R_t \sum_{s=0}^{T} Q_s a_{s,t} + R_t \sum_{s=0}^{T}(1-\theta_{s,t}^e)\left[\sum_{i=0}^{T-1} \frac{Q_s}{P}(Q_i - Q_{i+1})a_{i+1,t} + Q_T b_{s,t}\right] \\
&\quad -K_{t+1} - D_{t+1} \\
&\quad +D_{t+1} - R_t D_t + \theta_t^l w_t N \\
&\quad +R_t \sum_{s=0}^{T} \theta_{s,t}^e \left[\sum_{i=0}^{T-1} \frac{Q_s}{P}(Q_i - Q_{i+1})a_{i+1,t} + Q_T b_{s,t}\right]
\end{aligned}$$

$$
\begin{aligned}
C_t + G_t &= w_t N + R_t \sum_{s=0}^{T} Q_s a_{s,t} + R_t \sum_{s=0}^{T} \left[\sum_{i=0}^{T-1} \frac{Q_s}{P} (Q_i - Q_{i+1}) a_{i+1,t} + Q_T b_{s,t} \right] \\
&\quad - K_{t+1} - R_t D_t \\
&= w_t N + R_t \sum_{s=0}^{T} Q_s a_{s,t} + R_t \sum_{s=0}^{T} \frac{Q_s}{P} \sum_{i=0}^{T-1} (Q_i - Q_{i+1}) a_{i+1,t} + Q_T R_t a_{T+1,t} \\
&\quad - K_{t+1} - R_t D_t \\
&= w_t N + R_t \sum_{s=1}^{T} Q_s a_{s,t} + R_t \sum_{i=0}^{T-1} (Q_i - Q_{i+1}) a_{i+1,t} + Q_T R_t a_{T+1,t} \\
&\quad - K_{t+1} - R_t D_t \\
&= w_t N + R_t \sum_{s=1}^{T} Q_s a_{s,t} + R_t \sum_{i=0}^{T-1} Q_i a_{i+1,t} - R_t \sum_{s=1}^{T} Q_s a_{s,t} + Q_T R_t a_{T+1,t} \\
&\quad - K_{t+1} - R_t D_t \\
&= w_t N + R_t \sum_{i=0}^{T} Q_i a_{i+1,t} - K_{t+1} - R_t D_t \\
&= w_t N + R_t K_t - K_{t+1} \\
&= Y_t + (1 - \delta) K_t - K_{t+1}
\end{aligned}
$$

$$
C_t + I_t + G_t = Y_t
$$

Thus the income-expenditure identity holds, and the model is set up correctly.

B Solving the Analytic Case

Households maximize

$$
U = \ln c_0 + \beta \left[\ln c_1 + \rho \ln \left(R \left[(1 - \theta_0^e) b_0 \right]^{\mu_0} \left[(1 - \theta_1^e) b_1 \right]^{\mu_1} \right) \right], \tag{29}
$$

where $\mu_0 + \mu_1 = 1$, subject to

$$
c_0 + a_1 = (1 - \theta^l) w + B_0 \tag{30}
$$

$$
c_1 + a_2 = R a_1 + B_1 \tag{31}
$$

$$
b_0 + b_1 = a_2. \tag{32}
$$

The production function is

$$Y = K^{\alpha} \tag{33}$$

where the capital stock is

$$K = a_1 + a_2. \tag{34}$$

Estate tax revenue is

$$\Theta^e = R\left[\theta_0^e b_0 + \theta_1^e b_1\right]. \tag{35}$$

Labor tax revenue is

$$\Theta^l = \theta^l w. \tag{36}$$

The bequest balance equations are

$$B_s = (1-\theta_0^e)Rb_s. \tag{37}$$

The government's budget constraint is

$$G = \Theta^e + \Theta^l. \tag{38}$$

The Lagrangian is

$$\begin{aligned}\mathcal{L} &= \ln c_0 + \beta\left[\ln c_1 + \rho \ln\left(R\left[(1-\theta_0^e)b_0\right]^{\mu_0}\left[(1-\theta_1^e)b_1\right]^{\mu_1}\right)\right] \\ &\quad + \lambda_0[(1-\theta^l)w + B_0 - c_0 - a_1] + \lambda_1[Ra_1 + B_1 - c_1 - a_2] + \lambda_2[a_2 - b_0 - b_1].\end{aligned} \tag{39}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_0} = \frac{1}{c_0} - \lambda_0 = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = \frac{\beta}{c_1} - \lambda_1 = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial b_0} = \frac{\beta\eta\rho\mu_0}{b_0} - \lambda_2 = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial b_1} = \frac{\beta\eta\rho\mu_1}{b_1} - \lambda_2 = 0 \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial a_1} = -\lambda_0 + \lambda_1 R = 0 \tag{44}$$

and

$$\frac{\partial \mathcal{L}}{\partial a_2} = -\lambda_1 + \lambda_2 = 0. \tag{45}$$

$$\lambda_0 = \lambda_1 R = \lambda_2 R$$

$$\frac{1}{c_0} = \frac{\beta R}{c_1}$$

Thus we have the Euler equation

$$c_1 = \beta R c_0 \tag{46}$$

$$\frac{\mu_0}{b_0} = \frac{\mu_1}{b_1}$$

$$b_1 = \frac{\mu_1}{\mu_0} b_0$$

$$b_0 + b_1 = a_2$$

$$b_0 + \frac{\mu_1}{\mu_0} b_0 = a_2$$

$$b_0 = \frac{a_2}{1 + \frac{\mu_1}{\mu_0}} = \mu_0 a_2$$

$$b_1 = \mu_1 a_2$$

$$\frac{\beta}{c_1} = \frac{\beta \rho \mu_0}{b_0}$$

$$c_1 = \frac{1}{\rho} \frac{b_0}{\mu_0} = \frac{1}{\rho} a_2$$

$$a_2 = \rho c_1 = \beta \rho R c_0$$

$$c_1 + b_0 + b_1 = R a_1 + B_1$$

$$c_0 + (1 - \theta^l) w + a_1 = (1 - \theta^l) w + B_0$$

The lifetime budget constraint is

$$c_0 + (1 - \theta^l) w + \frac{c_1 + b_0 + b_1}{R} = (1 - \theta^l) w + B_0 + \frac{B_1}{R}$$

$$c_0 + \frac{\beta R c_0 + \beta \rho R c_0}{R} = (1 - \theta^l) w + B_0 + \frac{B_1}{R}$$

$$[1+\beta(1+\rho)]\, c_0 = (1-\theta^l)w + B_0 + \frac{B_1}{R}$$

$$c_0 = \frac{\eta}{1+\beta\eta(1+\rho)}\left[(1-\theta^l)w + B_0 + \frac{B_1}{R}\right] \tag{47}$$

$$c_1 = \frac{\beta R}{1+\beta(1+\rho)}\left[(1-\theta^l)w + B_0 + \frac{B_1}{R}\right] \tag{48}$$

$$b_0 = \frac{\mu_0\beta\rho R}{1+\beta(1+\rho)}\left[(1-\theta^l)w + B_0 + \frac{B_1}{R}\right] \tag{49}$$

$$b_1 = \frac{\mu_1\beta\rho R}{1+\beta(1+\rho)}\left[(1-\theta^l)w + B_0 + \frac{B_1}{R}\right] \tag{50}$$

The equilibrium capital stock is

$$K = a_1 + a_2$$

$$a_1 = (1-\theta^l)w(1-l) + B_0 - c_0$$
$$a_2 = \beta\rho R c_0$$

$$K = (1-\theta^l)w + B_0 + \frac{(\beta\rho R-1)}{1+\beta(1+\rho)}\left[(1-\theta^l)w + B_0 + \frac{B_1}{R}\right] \tag{51}$$

Define the MPC for young consumption as

$$m_0 = \frac{\eta}{1+\beta\eta(1+\rho)},$$

so

$$c_0 = m_0 H.$$

$$c_1 = m_0\beta R H$$

$$b_0 = m_0\mu_0\beta\rho R H$$
$$b_1 = m_0\mu_1\beta\rho R H$$

$$K = (1-\theta^l)w + (1-\theta_0^e)Rb_0 + \frac{\beta\rho R-1}{1+\beta(1+\rho)}\left[(1-\theta^l)w + (1-\theta_0^e)Rb_0 + (1-\theta_1^e)b_1\right]$$

In the analytic case, the bequest balance equations (37) reduce to

$$B_s = (1-\theta_s^e)Rb_s \tag{52}$$

for $s = 0, 1$. In the analytic case, we also have

$$W = (1-\theta^l)w + B_0 + \frac{B_1}{R} = (1-\theta^l)w + (1-\theta_0^e)Rb_0 + (1-\theta_1^e)b_1. \tag{53}$$

Meanwhile estate tax revenue is

$$\Theta^e = R(\theta_0^e b_0 + \theta_1^e b_1) \tag{54}$$

and labor tax revenue is

$$\Theta^l = \theta^l w. \tag{55}$$

Thus in equilibrium we must have

$$G = \theta^l w + R(\theta_0^e b_0 + \theta_1^e b_1),$$

and we can rewrite total lifetime wealth as

$$W = w + Rb_0 - G + (1 + (R-1)\theta_1^e)b_1.$$

$$\left[1 - \frac{\mu_0\beta\rho R^2}{1+\beta(1+\rho)} - (1+(R-1)\theta_1^e)\frac{\mu_1\beta\rho R}{1+\beta(1+\rho)}\right] W = w - G$$

$$W = \frac{w-G}{1 - \left[\mu_0 R - (1+(R-1)\theta_1^e)\mu_1\right]\frac{\beta\rho R}{1+\beta(1+\rho)}} \tag{56}$$

$$G = \theta^l w + \theta_0^e Rb_0 + \theta_1^e Rb_1$$

$$b_0 = m_0\mu_0\beta\rho RH$$
$$b_1 = m_0\mu_1\beta\rho RH$$

$$\Theta^e = (\mu_0\theta_0^e + \mu_1\theta_1^e)m_0\beta\rho R^2 H$$

$$H = \frac{w-G}{1 - \left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}$$

$$G = \theta^l w + \frac{(\mu_0\theta_0^e + \mu_1\theta_1^e)m_0\beta\rho R^2(w-G)}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}$$

$$w - G = (1-\theta^l)w - \frac{(\mu_0\theta_0^e + \mu_1\theta_1^e)m_0\beta\rho R^2(w-G)}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}$$

$$\left[1 + \frac{(\mu_0\theta_0^e + \mu_1\theta_1^e)m_0\beta\rho R^2}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}\right](w-G) = (1-\theta^l)w$$

$$\frac{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0 + (\mu_0\theta_0^e + \mu_1\theta_1^e)m_0\beta\rho R^2}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$\frac{1+\left((\mu_0\theta_0^e + \mu_1\theta_1^e)R - \left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\right)\beta\rho R m_0}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$\frac{1+\left(\mu_0(\theta_0^e-1)R + \mu_1\left(\theta_1^e R - (1+(R-1)\theta_1^e)\right)\right)\beta\rho R m_0}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$\frac{1+\left(\mu_0(\theta_0^e-1)R + \mu_1\left(\theta_1^e R - 1 - R\theta_1^e + \theta_1^e\right)\right)\beta\rho R m_0}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$\frac{1+\left(\mu_0(\theta_0^e-1)R + \mu_1\left(\theta_1^e - 1\right)\right)\beta\rho R m_0}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$\mu_s(1-\theta_s^e) = \nu_s$$

$$\overline{\nu} = \nu_0 + \frac{\nu_1}{R}$$

$$\frac{1-\beta\rho R^2 m_0\overline{\nu}}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}(w-G) = (1-\theta^l)w$$

$$w - G = \frac{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}{1-\beta\rho R^2 m_0\overline{\nu}}(1-\theta^l)w$$

$$\frac{K}{w-G} = 1+m_0(\beta\rho R-1)+\frac{\left(1+m_0(\beta\rho R-1)\right)\left(\mu_0+\theta_1^e\mu_1\right)R + m_0(\beta\rho R-1)(1-\theta_1^e)\mu_1}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}m_0\beta\rho R$$

$$\begin{aligned}&\frac{1-\beta\rho R^2 m_0\overline{\nu}}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}\frac{K}{(1-\theta^l)w}\\ &= 1+m_0(\beta\rho R-1)+\frac{\left(1+m_0(\beta\rho R-1)\right)\left(\mu_0+\theta_1^e\mu_1\right)R + m_0(\beta\rho R-1)(1-\theta_1^e)\mu_1}{1-\left[\mu_0 R + (1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}m_0\beta\rho R\end{aligned}$$

$$
\begin{aligned}
w &= (1-\alpha)K^{\alpha} \\
R &= \alpha K^{\alpha-1} \\
\frac{K}{w} &= \frac{K}{(1-\alpha)K^{\alpha}} = \frac{K^{1-\alpha}}{1-\alpha} = \frac{\alpha}{1-\alpha}\frac{1}{\alpha K^{\alpha-1}} = \frac{\alpha}{1-\alpha}\frac{1}{R}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{1-\beta\rho R^2 m_0\overline{\nu}}{1-\left[\mu_0 R+(1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0}\frac{1}{1-\theta^l}\frac{\alpha}{1-\alpha}\frac{1}{R} \\
&= 1+m_0(\beta\rho R-1)+\frac{(1+m_0(\beta\rho R-1))\left(\mu_0+\theta_1^e\mu_1\right)R+m_0(\beta\rho R-1)(1-\theta_1^e)\mu_1}{1-\left[\mu_0 R+(1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0} m_0\beta\rho R
\end{aligned}
$$

The right-hand side is

$$
\begin{aligned}
&[1+m_0(\beta\rho R-1)]\left[1-\left[\mu_0 R+(1+(R-1)\theta_1^e)\mu_1\right]\beta\rho R m_0\right] \\
&+\left[(1+m_0(\beta\rho R-1))\left(\mu_0+\theta_1^e\mu_1\right)R+m_0(\beta\rho R-1)(1-\theta_1^e)\mu_1\right] m_0\beta\rho R \\
&= [1+m_0(\beta\rho R-1)]\left[1+\left[\left(\mu_0+\theta_1^e\mu_1\right)R-\left[\mu_0 R+(1+(R-1)\theta_1^e)\mu_1\right]\right]\beta\rho R m_0\right] \\
&+(\beta\rho R-1)(1-\theta_1^e)\mu_1 m_0^2\beta\rho R \\
&= [1+m_0(\beta\rho R-1)]\left[1+\left[\mu_0 R+\theta_1^e\mu_1 R-\mu_0 R-\mu_1-\theta_1^e\mu_1 R+\theta_1^e\mu_1\right]\beta\rho R m_0\right] \\
&+(\beta\rho R-1)\nu_1 m_0^2\beta\rho R \\
&= [1+m_0(\beta\rho R-1)]\left[1+\left[-\mu_1+\theta_1^e\mu_1\right]\beta\rho R m_0\right]+(\beta\rho R-1)\nu_1 m_0^2\beta\rho R \\
&= [1+m_0(\beta\rho R-1)][1-\beta\rho R m_0\nu_1]+(\beta\rho R-1)\nu_1 m_0^2\beta\rho R \\
&= 1-\beta\rho R m_0\nu_1+m_0(\beta\rho R-1) \\
&= 1-m_0+m_0\beta\rho R(1-\nu_1)
\end{aligned}
$$

Thus the equation simplifies to

$$
\frac{1}{1-\theta^l}\frac{\alpha}{1-\alpha}\left(1-\beta\rho R^2 m_0\overline{\nu}\right) = \left[1-m_0+m_0\beta\rho R(1-\nu_1)\right]R,
$$

which in fact is quadratic in R. Note however that we need to simultaneously solve the government budget constraint if we hold G fixed.

$$
\frac{1}{1-\theta^l}\frac{\alpha}{1-\alpha}\left(1-\beta\rho m_0\left(\nu_0 R^2+\nu_1 R\right)\right) = \left[1-m_0+m_0\beta\rho R(1-\nu_1)\right]R
$$

Let

$$\sigma_l = \frac{1}{1-\theta^l}\frac{\alpha}{1-\alpha}$$

$$\sigma_l - \left[1 - m_0 + \beta\rho m_0 \sigma_l \nu_1\right] R - \left[m_0 \beta\rho(1-\nu_1) + \beta\rho m_0 \sigma_l \nu_0\right] R^2 = 0$$

$$C_R = \sigma_l > 0$$
$$B_R = -\left[1 - m_0 + \beta\rho m_0 \sigma_l \nu_1\right] < 0$$
$$A_R = -m_0\beta\rho\left[1 - \nu_1 + \sigma_l \nu_0\right] < 0$$

Thus we only have one positive root.

$$R = \frac{-B_R - \sqrt{B_R^2 - 4A_R C_R}}{2A_R} = \frac{\sqrt{B_R^2 + 4\left|A_R\right| C_R} - \left|B_R\right|}{2\left|A_R\right|}$$

$$A_R R^2 + B_R R + C_R = 0$$

$$\frac{\partial A_R}{\partial \theta_0^e} R^2 + 2RA_R \frac{\partial R}{\partial \theta_0^e} + B_R \frac{\partial R}{\partial \theta_0^e} = 0$$

$$\frac{\partial R}{\partial \theta_0^e} = -\frac{\frac{\partial A_R}{\partial \theta_0^e} R^2}{2RA_R + B_R}$$

$$\frac{\partial A_R}{\partial \theta_0^e} = m_0 \beta\rho\sigma_l \mu_0$$

$$\frac{\partial R}{\partial \theta_0^e} = -\frac{m_0\beta\rho\sigma_l\mu_0 R^2}{2RA_R + B_R} > 0.$$

$$\frac{\partial A_R}{\partial \theta_1^e} R^2 + 2RA_R \frac{\partial R}{\partial \theta_1^e} + B_R \frac{\partial R}{\partial \theta_1^e} + \frac{\partial B_R}{\partial \theta_1^e} R = 0$$

$$\frac{\partial R}{\partial \theta_1^e} = -\frac{\frac{\partial A_R}{\partial \theta_0^e} R^2 + \frac{\partial B_R}{\partial \theta_1^e} R}{2RA_R + B_R}$$

$$B_R = -\left[1 - m_0 + \beta\rho m_0 \sigma_l \nu_1\right] < 0$$
$$A_R = -m_0\beta\rho\left[1 - \nu_1 + \sigma_l \nu_0\right] < 0$$

$$\frac{\partial B_R}{\partial \theta_1^e} = \beta\rho m_0 \sigma_l \mu_1$$

$$\frac{\partial A_R}{\partial \theta_1^e} = -m_0 \beta \rho \mu_1$$

$$\frac{\partial R}{\partial \theta_1^e} = \frac{m_0 \beta \rho \mu_1 R(R - \sigma_l)}{2RA_R + B_R}$$

This is of ambiguous sign but will be negative if $R > \sigma_l$, which it is for our baseline case.